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Parallel Computing in Time-Frequency Distributions for Doppler Ultrasound Blood Flow Instrumentation

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ABSTRACT

Doppler blood flow spectral estimation is a technique for non-invasive cardiovascular disease detection. Blood flow velocity and disturbance may be determined by measuring the spectral mean frequency and bandwidth respectively. Typical methods for spectral estimation utilize Fourier Transform-based algorithms to estimate the spectral response of a signal. This current practice suffers from poor frequency resolution when estimating non-stationary signals.The work presented here describes some alternative methods based on time-frequency distributions from a Cohen´s class point of view. Four distribution cases are evaluated: Wigner Ville, Choi Williams, Bessel and Born Jordan. A discrete distribution is formulated for each case and a criterion for determining the interaction between the spectral components of the signal is also given. Simplified discretised expressions for the implementation of distributions are formulated, these leading to a reduction of the computations realized when comparing to original definitions. A general parallel approach for the computation of the distributions is proposed, implemented and assessed using a parallel DSP-based system. Results are applied to the development of a real-time spectrum analyser for Doppler blood flow instrumentation.

Key words:
Parallel Computing, Time-Frequency Distributions, Spectral Estimation, Doppler Ultrasound, Blood Flow Instrumentation.

RESUMEN

La estimación espectral del la señal Doppler generada por el flujo sanguíneo, es una técnica no-invasiva utilizada para la detección de padecimientos cardiovasculares. La velocidad del flujo sanguíneo así como la turbulencia pueden ser determinados mediante la medición de la frecuencia espectral media y el ancho de banda respectivamente. Los métodos típicos de estimación espectral utilizan algoritmos basados en la Transformada de Fourier para estimar la respuesta espectral de una señal. Esta práctica común adolece de una limitada resolución en frecuencia cuando se utiliza en señales no-estacionarias. El trabajo presentado aquí describe algunos métodos alternativos basados en distribuciones tiempo-frecuencia desde el punto de vista de la clase de Cohen.

Cuatro casos de distribución son evaluados: Wigner-Ville, Choi Williams, Bessel and Born Jordan. Se ha formulado un distribución discreta para cada caso y se propone un criterio para determinar la interacción entre las componentes espectrales de la señal. Se han formulado expresiones discretisadas simplificadas para la implementación de las distribuciones, lo cual ha resultado en una reducción en la complejidad computacional comparada con las definiciones originales. Una estrategia general para realizar el cómputo paralelo de las distribuciones es propuesto, implementado y evaluado utilizando un sistema paralelo basado en una arquitectura de procesadores digitales de señales (DSP´s). Los resultados son aplicados al desarrollo de un analizador de espectros en tiempo real para una aplicación de instrumentación Doppler ultrasónica de flujo sanguíneo.

Palabras clave:
Cómputo paralelo, Distribuciones tiempo-frecuencia, Estimación espectral, Ultrasonido Doppler, Flujometría sanguínea.

INTRODUCTION

A pulsed Doppler blood flow detector is an useful device employed for the diagnostic and monitoring of cardiovascular disease progression and treatment. It can determine the blood velocity and detect flow disturbances by measuring the Doppler shift in frequency of ultrasound scattered from the blood flow. An increase in the range of Doppler shift frequencies, as a result of flow disturbance, can be used to detect atherosclerotic lesions in arteries in an early stage. As the velocity of blood flow within arteries is periodic the Doppler spectrum alters not only in mean frequency but also in shape throughout each cardiac cycle. Therefore, the Doppler signal is a cyclo-stationary Gaussian stochastic signal and can be considered as quasi-stationary over short time segments (2-20 ms) [1]. Conventional methods for real-time spectral analysis use the Fast Fourier Transform (FFT) on consecutive or overlapping time segments, but this current practice suffers from poor frequency resolution as a result of the time segment duration and non-stationarity[2]. Other types of spectral estimators, called time-frequency distributions, have been developed. Unlike conventional methods these distributions are not limited to the use of stationary signals[6]. Despite of this important advantage, the number of calculations involved in obtaining the spectral estimation increases substantially compared to the traditional methods.

Therefore, it is desirable to simplify the formulation of the distributions in such a way that the computations involved can be reduced without any loss in the spectral resolution. On the other hand, there are a great variety of time-frequency distributions. It would be very useful to develop an analysis criterion such that can provide a tool for selecting the optimum time-frequency distribution according to the features of the signal under consideration. A general parallel approach for the computation of the distributions is proposed, implemented and assessed using a parallel DSP-based system. Results are applied to the development of a real-time spectrum analyser for Doppler blood flow instrumentation [11, 12].

TIME-FREQUENCY DISTRIBUTIONS

This section formulates the so-called Cohen's class for the time-frequency distributions and it defines some related concepts.

The Cohen's Class

The Cohen´s class in terms of time frequency distributions [6] can be formulated as follows. Let the time-frequency distribution kernel be defined as $\phi(\theta,\tau)$. This kernel will define the particular characteristics of each time-frequency distribution. Then, the Cohen's class for the time-frequency distributions with kernel $\phi(\theta,\tau)$ can be defined as:

$$TFD(t,\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\phi(\theta,\tau)e^{-j\theta(t-\mu)}d\theta\right]x\left(\mu+\frac{\tau}{2}\right)x^{*}\left(\mu-\frac{\tau}{2}\right)d\mu\right]e^{-j\omega\tau}d\tau \quad (1)$$

Auto-term, Crossing Term and Crossing Term Weighting Factor

In order to establish a comparison criterion between the different time-frequency distributions considered in this paper, it is necessary to develop a method that may determine the degree in which the various components of a signal interact when the time frequency distribution is calculated[4]. Consider the following signal, which is composed of a finite number of sinusoidal signals with constant amplitude A_n, frequency ω_n and phase θ_n:

$$x(t)= \sum_{n=1}^{N} A_n e^{j(\omega_n t+\theta_n)} \tag{2}$$

The n components of the signal in eq. (2) interact between them through eq. (1). The interactions of the components with themselves generate the so-called auto-terms of the distribution, which are always positive and constitute the spectral contents of the signal. On the other hand, interactions between different components generate the so-called crossing terms of the distribution, which can be positive or negative and are added to the spectral contents of the signal. Therefore it is desirable to minimize such terms.

Substituting eq. (2) in eq. (1) and grouping the auto-terms in the first summation and crossing terms in the second summation, a time-frequency distribution can be expressed as:

$$TFD(t,\omega)= 2\pi\sum_{n=1}^{N} A_n^2 \delta(\omega-\omega_n) + \sum_{n=1}^{N}\sum_{\substack{m=1\\ n\neq m}}^{N} F_{TFD}(\omega) A_n A_m \cos((\omega_n-\omega_m)t+\theta_n-\theta_m) \tag{3}$$

where $F_{TFD}(w)$ is the crossing terms weighting factor, a quantitative measure for evaluating the different time-frequency distributions. The following sections describe the distributions according to their definition.

The Wigner Ville Distribution

According to its definition[4, 7, 9], the Wigner Ville distribution for the discrete case is given by:

$$DWVD(n,k)= 2\sum_{\tau=-N+1}^{N-1} W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}} x(n+\tau)x^*(n-\tau) \tag{4}$$

where n represents the discrete time and k the discrete frequency; both variables are normalized.

The Choi Williams Distribution

According to its definition[4, 5], the Choi Williams distribution for the discrete case is given by eq. (5), where $\sigma>0$ is a scaling factor.

$$CWD(n,k)= = 2\sum_{\tau=-N+1}^{N-1} W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}} \sum_{\mu=-M}^{M}\sqrt{\frac{1}{4\pi\tau^2/\sigma}}\, e^{-\frac{\mu^2}{4\tau^2/\sigma}} x(\mu+n+\tau)x^*(\mu+n-\tau) \tag{5}$$

The Bessel Distribution

According to its definition[4, 8], the Bessel distribution for the discrete case is given by eq. (6), where $\alpha>0$ is a scaling factor.

$$BDB(n,k)= = 2\sum_{\tau=-N+1}^{N-1} W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}} \sum_{\mu=-2\alpha|\tau|}^{2\alpha|\tau|}\frac{1}{\pi\alpha|\tau|}\sqrt{1-\left(\frac{\mu}{2\alpha\tau}\right)^2}\, x(\mu+n+\tau)x^*(\mu+n-\tau) \tag{6}$$

The Born Jordan Distribution

According to its definition[6], the Born Jordan distribution for the discrete case is given by (7), where $a>0$ is a scaling factor.

$$DBJD(n,k)= = 2\sum_{\tau=-N+1}^{N-1} W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}} \sum_{\mu=-2\alpha|\tau|}^{2\alpha|\tau|}\frac{1}{4\alpha|\tau|} x(\mu+n+\tau)x^*(\mu+n-\tau) \tag{7}$$

EVALUATION OF THE TIME-FREQUENCY DISTRIBUTIONS BASED ON THE CROSSING TERMS WEIGHTING FACTOR

As stated previously, an ideal crossing terms weighting factor would be one that eliminates the crossing terms. But, a desirable and more realistic situation would be that the weighting factor concentrated the crossing terms due to two different frequency components of the signal around such frequencies and not around other frequencies or spread them out over a wide range of frequencies.

Substituting the signal defined by eq. (2) in eq. (1) and arranging the crossing terms in the distributions according to eq. (3), the crossing terms weighting factors of each distribution are obtained and defined by the following expressions

For the Wigner Ville distribution:

$$F_{WVD}(\omega) = 2\pi\delta\left(\omega - \frac{\omega_n + \omega_m}{2}\right) \quad (8)$$

For the Choi Williams distribution:

$$F_{CWD}(\omega) = \sqrt{\frac{\pi\sigma}{(\omega_n - \omega_m)^2}}\, e^{-\frac{\sigma}{4(\omega_n - \omega_m)^2}\left(\omega - \frac{\omega_n + \omega_m}{2}\right)^2} \quad (9)$$

For the Bessel distribution:

$$F_{BD}(\omega) = \frac{4}{\alpha|\omega_n - \omega_m|} U_0\left(\frac{\omega - \frac{\omega_n + \omega_m}{2}}{\alpha(\omega_n - \omega_m)}\right)\sqrt{1 - \left(\frac{\omega - \frac{\omega_n + \omega_m}{2}}{\alpha(\omega_n - \omega_m)}\right)^2} \quad (10)$$

For the Born Jordan distribution:

$$F_{BJD}(\omega) = \frac{\pi}{\alpha|\omega_n - \omega_m|} P_{2\alpha(\omega_n - \omega_m)}\left(\omega - \frac{\omega_n + \omega_m}{2}\right) \quad (11)$$

where $P\alpha(t)$ is a rectangular symmetrical pulse of duration α. Figure 1 shows a global view of the crossing terms weighting factors for the Wigner Ville, Choi Williams, Bessel and Born Jordan distributions. The weighting factor is defined in terms of ω and contains the interacting frequencies ω_n and ω_m defined in signal in eq. (2). Such frequencies can be added or subtracted depending on the behavior of each distribution. Consider a normalized addition, that is $\omega_n+\omega_m=1$, then, the graphs relate the weighting factor versus ω_n and ω, where $0<\omega_n<1$. Given the normalized addition, for each ω_n value, ω_m is given by $1-\omega_m$.

ANALYSIS

Figures 2 and 3 depict graphs which show the differences of the crossing terms weighting factors between Born Jordan minus Choi Williams, Born Jordan minus Bessel, Bessel minus Choi Williams and Wigner Ville minus any other distribution. The dark zones in the graphs correspond to points where the weighting factor of the first distribution under comparison is greater than the second one. For the purpose of this analysis sca-

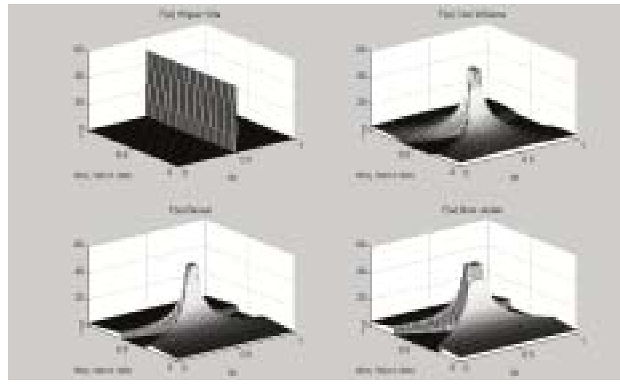

Figure 1. Global view of the crossing terms weighting factors for the Wigner Ville, Choi Williams *(σ=5),* Bessel *(α=2)* and Born Jordan *(α=2)* distributions.

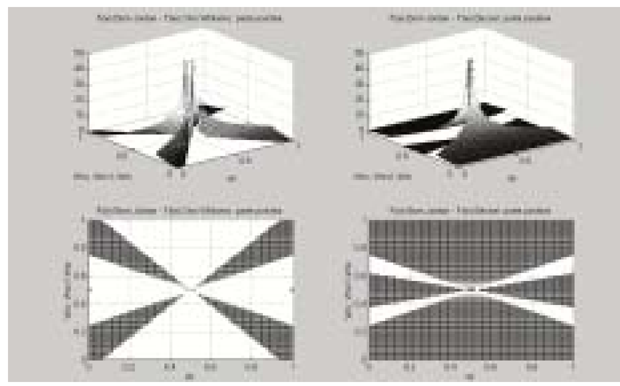

Figure 2. Differences of the crossing terms weighting factors between Born Jordan minus Choi Williams and Born Jordan minus Bessel distribution.

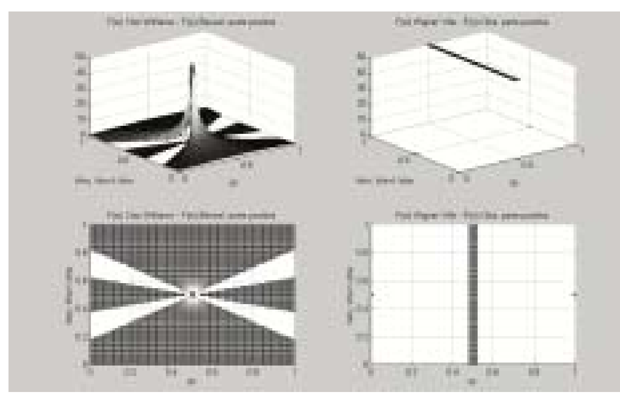

Figure 3. Differences of the crossing terms weighting factors between Choi Williams minus Bessel and Wigner Ville minus any other distribution.

ling factors $\sigma=5$, $\alpha=2$ and $\alpha=1$ are considered in the Choi Williams, Bessel and Born Jordan distributions, respectively.

In general, it is observed that the weighting factor for the Bessel distribution is smaller than the Choi William's and the Born Jordan's, and that the Choi Williams's is smaller than the Born Jordan's. These results indicate that the Bessel distribution spreads out the crossing terms better than the Choi Williams's and Born Jordan's distributions and, in consequence, estimates with more precision the spectral contents of a signal in the presence of noise. All this facts explains the reason why the Bessel distribution is less sensitive to the presence of noise[4]. But there is a direct relationship between the precision and the amount of calculations involved in the estimation of the spectral contents of the signal.

In the case of the Wigner Ville distribution, the crossing terms are concentrated on the average of such frequencies (due to the two frequency components of the signal). Therefore, this distribution estimates with better precision the spectral contents of noiseless signals with small bandwidth.

The figure 4 shows the normalized instantaneous frequency estimation error versus the time involved in performing the calculations, considering a simulated non-stationary Doppler signal proposed in[4], that is, a band limited stochastic signal with Gaussian probability density function which models a signal sampled at the center of a normal carotid artery.

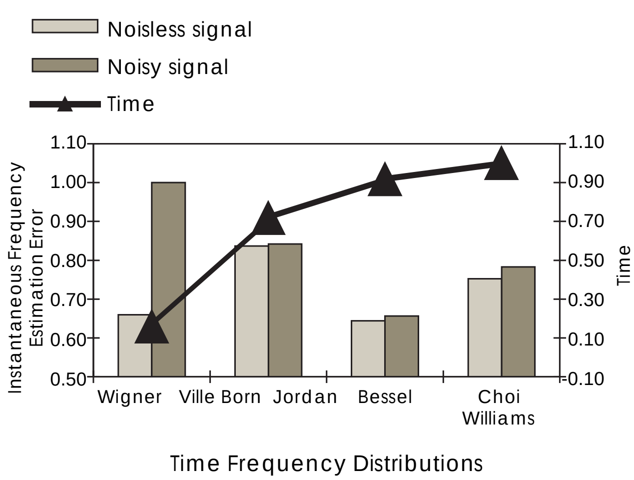

Figure 4. Normalized instantaneous frequency Estimation error and normalized executiontime of noiseless signal and noisy signal using Wigner Ville, Born Jordan, Bessel and Choi Williams time-frequency distributions.

It is important to point out that all the distributions are affected by a scaling factor which in turn modifies the crossing term weighting factors. As stated previously, the results have been obtained considering $\sigma=5$, $\alpha=2$ and $\alpha=1$ in the Choi Williams, Bessel and Born Jordan distributions, respectively. These optimum scaling factors have been found experimentally and they depend on the characteristics of the signal under study.

REDUCING THE COMPUTATIONAL COMPLEXITY OF THE DISCRETE DEFINITIONS

In order to evaluate the different distributions for spectral estimation, a discrete signal $x(n)$ is considered. Such a signal contains $2N-1$ elements, where N is a power of 2 and the element range is from $-N+1$ to $N-1$, therefore $x(0)$ is the central element. Based on these elements, this section presents a reduction in computational terms of the number of calculations involved in the evaluation of each of the distributions considered in this paper.

The Wigner Ville Distribution

Considering eq. (4) for estimating the Wigner Ville distribution and evaluating it in $n=0$ [3, 7], an equivalent simplified expression would be given by:

$$DWVD(0,k) = 4\,\mathrm{Re}\left[\sum_{\tau=0}^{N-1} W(\tau)W^*(-\tau)\,e^{-j\frac{2\pi k\tau}{N}}\,x(\tau)x^*(-\tau)\right] - 2|x(0)|^2 \qquad (12)$$

Assuming that $W(\tau)W^*(-\tau)$ is a single factor then, for each value of κ in eq. (4) evaluated in $n=0$, there are $6N-3$ complex multiplications, $2N-2$ complex additions and 1 scalar multiplication, whereas in eq. (12) there are $3N+1$ complex multiplications, N complex additions and 2 scalar multiplications.

The Choi Williams Distribution

Similarly, considering eq. (5) for estimating the Choi Williams distribution and evaluating it in $n=0$, an equivalent simplified expression would be given by:

$$DCWD(0,k) = -2|x(0)|^2 + 4\,\mathrm{Re}\left[\sum_{\tau=0}^{N-1} W(\tau)W^*(-\tau)\,e^{-j\frac{2\pi k\tau}{N}} \sum_{\mu=-N+1+|\tau|}^{N-1-|\tau|} \sqrt{\frac{1}{4\pi\tau^2/\sigma}}\,e^{-\frac{\mu^2}{4\tau^2/\sigma}}\,x(\mu+\tau)x^*(\mu-\tau)\right] \qquad (13)$$

where the summation respect to m for $\tau=0$ is $x(0)x^*(0)$.

Assuming that $W(\tau)W^*(-\tau)$ and the square root multiplied by the exponential are single factors then, for each value of κ in eq. (5) evaluated in $n=0$, there are $8N^2-4N$ complex multiplications, $4N^2-6N+2$ complex additions and 1 scalar multiplication, whereas in eq. (13) there are $2N^2-2N+1$ complex multiplications, N^2-2N complex additions and 2 scalar multiplications.

The Bessel Distribution

Considering eq. (6) for estimating the Bessel distribution and evaluating it in $n=0$, an equivalent simplified expression would be given by:

$$DCWD(0,k)= -2|x(0)|^2 + 4\,\mathrm{Re}\left[\sum_{\tau=0}^{N-1} W(\tau)W^*(-\tau)\,e^{-j\frac{2\pi k\tau}{N}} \sum_{\mu=-N+1+|\tau|}^{N-1-|\tau|} \sqrt{\frac{1}{4\pi\tau^2/\sigma}}\,e^{-\frac{\mu^2}{4\tau^2/\sigma}}\,x(\mu+\tau)x^*(\mu-\tau)\right] \tag{14}$$

where the summation respect to μ for $\tau=0$ is $x(0)x^*(0)$.

Assuming that $W(\tau)W^*(-\tau)$ and the square root divided by $\pi\alpha|\tau|$ are single factors then, for each value of κ in eq. (6) evaluated in $n=0$, there are $8\alpha N^2-8\alpha N$ complex multiplications, $4\alpha N^2-4\alpha N-2N$ complex additions and 1 scalar multiplication, whereas in eq. (14) there are less than $4\alpha N^2-4\alpha N+1$ complex multiplications, less than $2\alpha N^2-2\alpha N-N$ and 2 scalar multiplications.

The Born Jordan Distribution

Considering eq. (7) for estimating the Born Jordan distribution and evaluating it in $n=0$, an equivalent simplified expression would be given by:

$$DBJD(0,k)= -2|x(0)|^2 + 4\,\mathrm{Re}\left[\sum_{\tau=0}^{N-1} W(\tau)W^*(-\tau)\,e^{-j\frac{2\pi k\tau}{N}} \sum_{\mu=max\{-2\alpha|\tau|,-N+1+|\tau|\}}^{min\{2\alpha|\tau|,N-1-|\tau|\}} \frac{1}{4\alpha|\tau|}\,x(\mu+\tau)x^*(\mu-\tau)\right] \tag{15}$$

where the summation respect to μ for $\tau=0$ is $x(0)x^*(0)$. The analysis is similar to Bessel's.

PARALLEL PROCESSING OF THE TIME-FREQUENCY DISTRIBUTIONS

As stated previously, the use of time-frequency distributions for the spectral estimation of signals opens the possibility of analyzing non-stationary signals. However, the computational cost is high. In view of this, this paper has proposed a reduction in the amount of calculations involved for evaluating the original definitions of each distribution, as developed in the previous section. In addition, it is proposed the use of parallel processing techniques to further reduce the time required to perform the evaluations. In particular, a pipeline scheme is used with three stages.

The first stage calculates the analytic signal $x_a(t)$ of the real signal. The second stage calculates the generalized time-indexed auto-correlation function $R'_x(t,\omega)$ for $t=0$ of $x_a(t)$, which is the integral respect to μ in eq. (1). Finally, the third stage calculates the Fourier transform of $R'_x(0,\omega)$, which is the time-frequency distribution $TFD(t,w)$ for $t=0$ of the real signal. Figure 5 shows the pipeline structure of the process. For the first and third stages the calculations are relatively simple and a Fast Fourier Transform (FFT) algorithm is used. However, the second stage requires of a more complex process, therefore this stage is further exploited using for this purpose a parallel farm computational model in a star topology.

Here, each node calculates a set of operations of the generalized time-indexed auto-correlation function. Although the expressions for the evaluation of each of the time-frequency distributions are different, this second stage can be adapted easily adding or subtracting processors according to the needs.

A system based on the ADSP-21062 SHARC has been used for implementing the modified covariance algorithm. The ADSP-21062 SHARC is a high-performance signal processor using a Super Harvard Architecture (i.e. four independent buses for

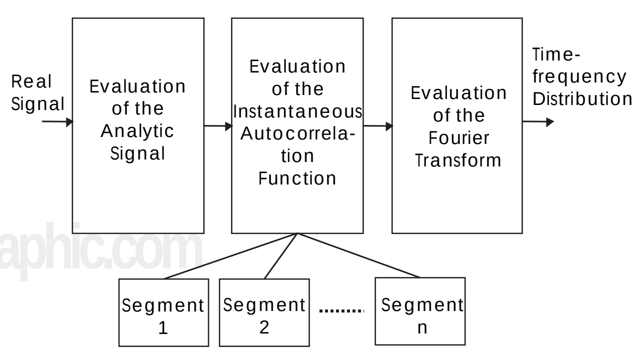

Figure 5. General Parallel Processing scheme for the evaluation of the time-frequency distributions.

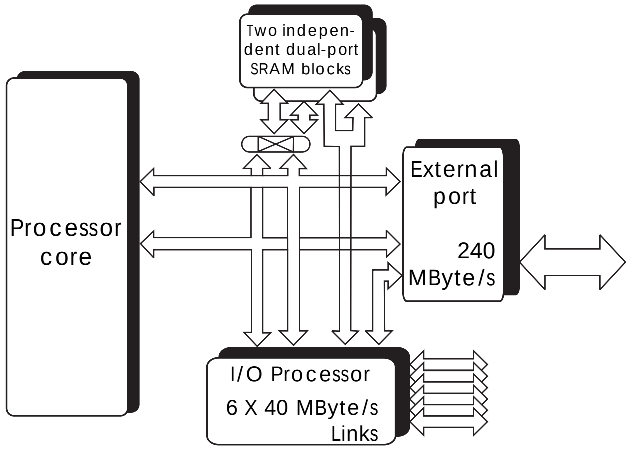

Figure 6. ADSP-21060/62 SHARC block diagram.

dual-data, instructions and I/O), see Figure 6. It integrates three 32-bit IEEE floating-point computation units (multiplier, ALU and shifter), a 2 Mbits dual port on-chip SRAM and multiprocessing features. It performs 40 MIPS, 120 MFLOPS peak and 80 MFLOPS sustained and 6 DMA communication links with a maximum bandwidth of 240 MB/sec [10].

RESULTS

Figure 7 shows the time performance of the Wigner Ville and the Bessel distributions for two different parallel architectures (pipeline and farm+pipeline). Note that the Choi Williams and the Born Jordan distributions behave close to the Bessel distribution.

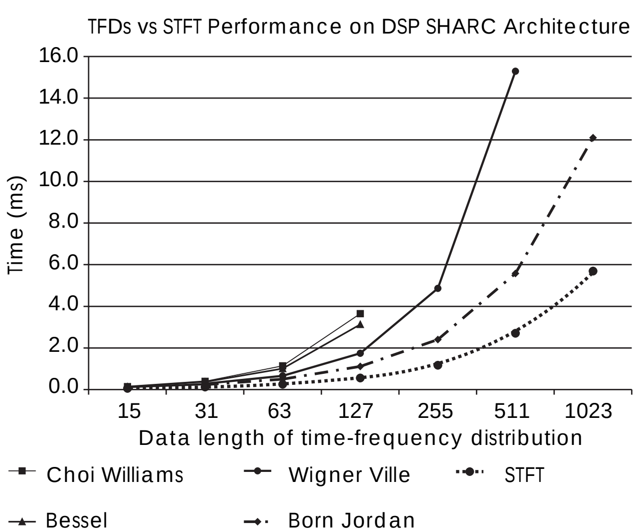

Figure 7. Execution time vs data length of Time-Frequency distribution and STFT implemented in a DSP-SHARC architecture.

A Doppler blood flow real-time spectral analyser is been developed based on the parallel DSP-system described. Screen-shots of spectrograms obtained for Carotid and Humeral Arteries are presented obtained by the system is shown in Figure 8.

CONCLUSIONS

Conventional methods for spectral estimation of Doppler ultrasounf signals are limited to the analysis of stationary signals to produce a good estimate, however, these methods offer poor resolution when dealing with non-stationary signals.

This paper has presented some alternative methods based on the so-called time-frequency distributions for spectral analysis. Four methods based on the Cohen's class have been analyzed,namely the Wigner Ville, the Choi Williams, the Bessel and the Born Jordan distributions. A comparison criterion based on the crossing terms weighting factor has been proposed showing that the Bessel distribution spreads out the crossing terms better than the

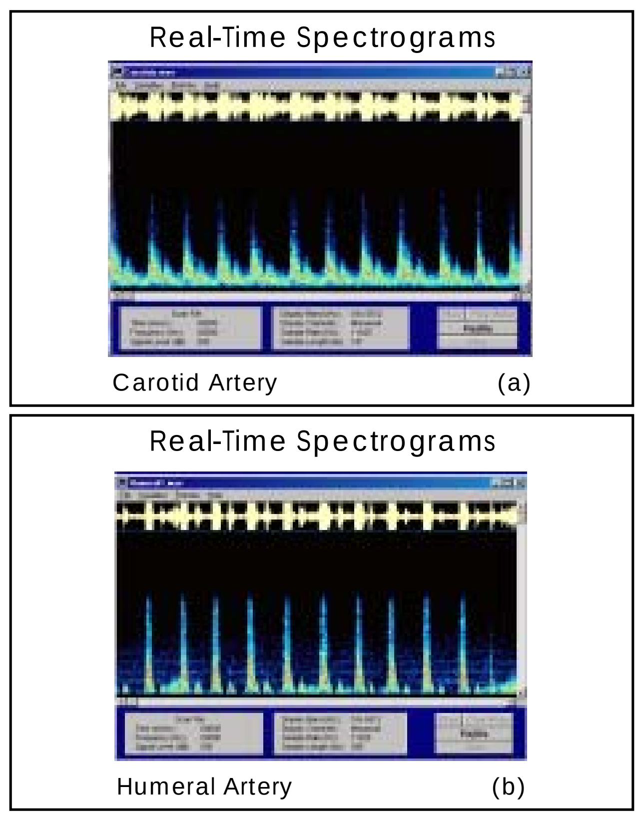

Figure 8. Real-time spectrograms for carotid (a) and humeral (b) arteries.

Choi Williams's and Born Jordan's distributions and, in consequence, estimates with more precision the spectral contents of a signal in the presence of noise, whereas the Wigner Ville distribution estimates with better precision the spectral contents of noiseless signals with small bandwidth. This analysis was conducted taking into account the optimum scaling factors: $\sigma=5$, $\alpha=2$ and $\alpha=1$ for the Choi Williams, Bessel and Born Jordan distributions, respectively. This work also has proposed a simplification in the complexity of the expressions utilized for calculating the time-frequency distributions giving as a result a reduction of at least half the operations involved in the original definition.

A parallel processing scheme for the computation of the time-frequency distribution methods has been implemented. Here, a pipeline scheme with three stages is utilized, corresponding to the second stage to deal with the more expensive computational process (evaluation of the generalized time-indexed auto-correlation function). A generalized scheme has been implemented which can be adapted easily according to the time-frequency distribution under consideration. As a result, the Wigner Ville distributions shows a better performance in a pipeline parallel arquitecture, while the Choi Williams, Bessel and Born Jordan distributions perform better in a farm+pipeline scheme.

Current work is addressed in the development of a Doppler blood flow real-time spectral analyser based on the parallel DSP-system, aiming to be applied for cardiovascular desease detecction.

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